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Turbulence characterization with a Shack-Hartmann wavefront sensor

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Hartmann sensor data can reveal rapid variation in optical-turbulence coherence length and produce estimates of the Greenwood frequency and inner scale.

In the field of optical atmospheric propagation, knowledge of optical-turbulence strength and other key statistical parameters is crucial for performance prediction and system design. Hartmann-sensor data can be used to reliably estimate the essential parameters characterizing optical turbulence, including the Fried coherence length (r_0), the Greenwood frequency (f_G, a measure of the temporal turbulence bandwidth due to wind or beam slewing), and the inner scale of turbulence (ℓ_0). The earliest approaches for estimating r_0 were based on modulation-transfer-function (MTF) measurements.[1] This requires accurate calibration and stability of the system MTF, which is often problematic. Astronomers have used differential motion (DIMM: differential image-motion monitor) and scintillation to measure 'seeing' conditions.[2] (In astronomy, seeing refers to the blurring of images caused by moving air cells in the Earth's atmosphere.) Others have used the slope-structure function estimated from a Hartmann wavefront sensor, principally for r_0 estimation.[3]

It is well known that $r_0^{-5/3}$ is proportional to the variances of quantities related to the optical phase. For example, one could estimate r_0 from the variance of the focus mode in aberration, the tilt, or the total phase. In general, a quite general form for an r_0 estimator is

$$r_0^{-5/3} = \sum_k X_k^2, \tag{1}$$

where X_k is some finite set of random variables, such as scaled Zernike coefficients or scaled slope measurements from a Hartmann sensor. These are scaled because the proportionality constants have been absorbed into the definition of each X_k. We have shown that the slope discrepancy[4,5] (or rotational component) of the slopes from a Hartmann wavefront sensor provides a very effective estimator of this general form. In this case, the vector $[X_k]$ consists of the x- and y-slope-discrepancy (or circulation) component of the measured Hartmann slopes. We developed a high-resolution, high-frame-rate, mobile sensor to use these estimation techniques (see Figure 1). It is mounted on a shock-isolation system inside a small trailer. The sensor consists of a 32×32 Hartmann array with 18×18 pixels^2 per subaperture, imaged through a 40cm telescope. This provides a large linear range for open-loop measurements. Data can be captured at a maximum rate of 8639 frames/s. This high data rate is important for temporal-structure-function and noise calculations.

Figure 1. *Mobile turbulence sensor.*

The estimation error from Equation (1) is readily obtained. If we assume that each X_k is a normal, random variable with zero mean, then $r_0^{-5/3}$ is a χ^2 random variable and the estimation error can be derived from the covariance of the vector process. The error will be inversely proportional to the statistical number of

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Table 1. *Number of statistical degrees of freedom for quantities commonly derived from Hartmann-sensor measurements, calculated for a 32×32 array of 700 Hartmann subapertures. g tilt: Tilt derived from gradient measurements produced by the subapertures. The phases and tilt angles are given in arbitrary units.*

Statistical quantity	Degrees of freedom
Reconstructed phase	2.58
Tilt-removed reconstructed phase	10.1
Sensor g tilts	19.1
Full aperture tilt-removed g tilts	96.9
Slope discrepancy	416
Differential tilt	832

degrees of freedom (NDOF). Table 1 lists the NDOFs for typical quantities derived from Hartmann-sensor measurements. The slope discrepancy and differential tilt (i.e., the rms difference of tilts in two adjacent subapertures) have very high NDOF values, resulting in r_0 estimation errors of less than 5%. Noise will reduce the NDOF of a given process, but noise-removal techniques can be applied to both differential tilt and slope discrepancy.

Figures 2 and 3 represent r_0 estimates at a wavelength of 633nm, based on differential tilt and slope discrepancy, for very high and moderate-to-weak levels of turbulence, respectively. The data in Figure 2 was taken over a dry creek bed with an optical path of 740m, while that in Figure 3 was collected in a parking lot with a range of 45m. In both cases, significant variation in r_0 levels are observed over relatively short times. Because of the small estimation errors for slope discrepancy and differential tilt, these large changes imply significant variation in the underlying turbulence strength. This type of variation was not atypical in our measurements.

The Greenwood frequency can be obtained from the temporal phase-structure function.[4] The structure function, $D_\Phi(\tau)$, for a spatial phase $\Phi(t)$ and small time separation, τ, is

$$D_\Phi(\tau) = \langle|\Phi(t+\tau) - \Phi(t)|^2\rangle = 28.4(f_G\tau)^{5/3}. \quad (2)$$

By reconstructing the phase from each frame, the phase-structure function can be estimated as a function of frame separation. Special care must be taken to handle the noise (see Figure 4). The raw, reconstructed phase-structure function does not exhibit the 5/3 power-law asymptote because of the effects of measurement noise. The noise (both signal noise and turbulence-measurement error caused by sensor-fitting error) can be estimated using the slope-discrepancy structure function. When this noise-structure function is subtracted from the reconstructed phase-structure function, the 5/3 power law emerges and f_G can be estimated from any point on this asymptote. For this data, $f_G = 9.8$Hz.

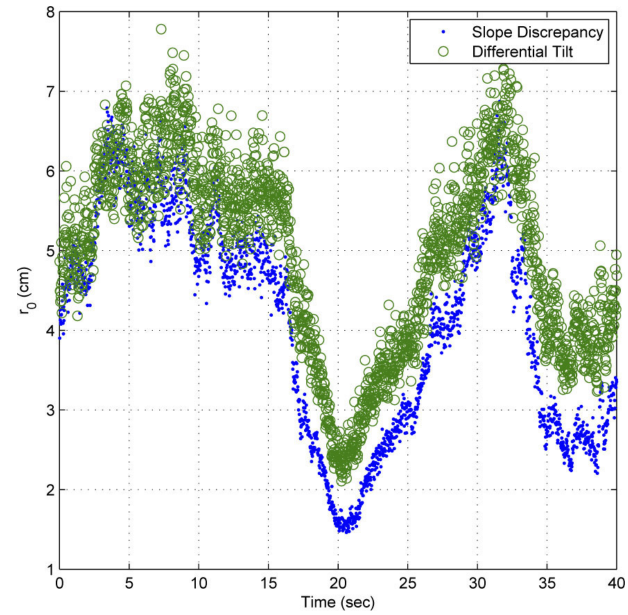

Figure 2. *Time-varying coherence length (r_0) for strong turbulence.*

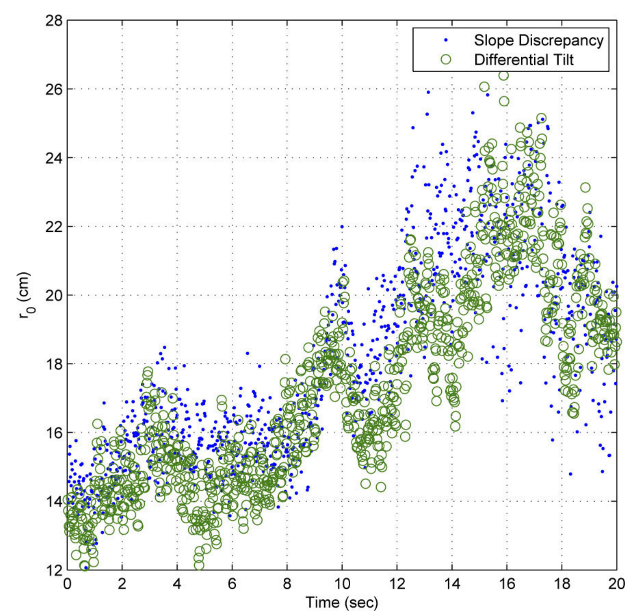

Figure 3. *Time-varying r_0 for moderate-to-weak turbulence.*

We also developed a technique for estimating the inner scale of turbulence based on the Hill-spectrum assumption.[6] This

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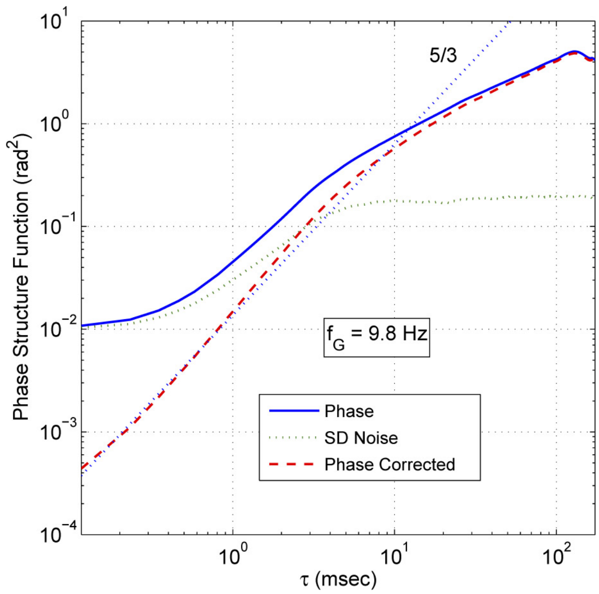

Figure 4. *Noise-corrected phase-structure function. f_G: Greenwood frequency. SD: Slope discrepancy. τ: Time separation.*

technique uses ratios of statistical quantities derived from the sensor data. Estimates of ℓ_0 from near-ground horizontal paths are between 3 and 5mm.

In summary, a high-spatial- and temporal-resolution Hartmann sensor can produce extremely reliable estimates of r_0 and f_G. Field results have illustrated rapid variation in r_0 on short timescales. This type of behavior will not be captured with an MTF or DIMM sensor. Knowledge of rapid temporal turbulence variation is significant for modeling of propagation and adaptive-optics performance. Modeling based on average r_0 values will not capture the rapid increases in turbulence that can produce strong performance fades. The field results to date have been limited to relatively short-range horizontal paths near the ground. We will next extend our experiments to a broader set of propagation conditions.

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